Novel Artificial Embedded Circuit Meta-Material for Design of Tunable Electro-Ferromagnetic Permeability Medium*

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Key Contribution: **Utilizing the available materials in nature, one can easily obtain a dielectric medium with almost any desired permittivity; however, the atoms and molecules of natural materials or their mixtures prove to be a rather restrictive set when one tries to achieve a desired permeability at a desired frequency, specially in gigahertz range [1]. The ability to design materials with both ε and μ parameters would represent tremendous progress in technology. The main challenge in this paper is to design a novel artificial periodic tunable embedded circuit meta-material to successfully present a medium with both dielectric (ε) and magnetic (μ) properties. The building block unit cell of the proposed structure is a miniaturized high *Q* tank circuit terminated to the ferro-electric material. The transmission line approach and FDTD technique are applied to successfully obtain the constitutive parameters of the periodic meta-material and highlight the unique characteristics of medium.**

I. Embedded Circuit ε-μ Meta-Material

The focus in this section is to obtain the concept and characteristics of the composite embedded circuit meta-material offering the permittivity and tunable permeability parameters.

A. Magnetic Property

The simplest form of EM waves in a homogeneous-source free material ($\varepsilon-\mu_0$) is a TEM plane wave. It is customary to view the homogeneous medium supporting the plane wave by a transmission line supporting the TEM wave having the same characteristic impedance and propagation constant as those of the medium. Equivalently the line inductance (L_l) and capacitance (C_l) per unit length are the same as the permittivity (ε) and permeability (μ_0) of the medium.

Now let us consider a modification to the conventional transmission line configuration L_l - C_l by inserting a thin wire loop tank circuit with the self-inductance L_p, resistance R_p, and lumped capacitor C_p, as depicted in Fig. 1(a). The magnetic flux linking the transmission line induces a current in the loop in a direction so that the magnetic flux generated by the loop would oppose the transmission line magnetic flux. Fig. 1(b) shows the equivalent circuit of the transmission line model. The mutual coupling between the loop and the transmission line inductance are denoted by mutual inductance M. Loading the transmission line in a periodic fashion with identical tank circuits creates a new transmission line with the equivalent inductance per unit length L_{eq}, which is readily obtained from the circuit model (Fig. 1(b)). The corresponding medium of the transmission line has then the effective permeability μ_{eff} ($\mu_{eff} \leftrightarrow L_{eq}$, $\mu_0 \leftrightarrow L_l$)

$$\mu_{eff} = \mu_0 \left(1 - \kappa^2 \frac{1}{1 - \omega_p^2/\omega^2 - j/Q} \right) \tag{1}$$

where,

$$\omega_p = 1\Big/\sqrt{L_p C_p}\ ,\ \kappa = M\Big/\sqrt{(L_l \Lambda_x) L_p}$$

$$L_l = \mu_0 \Lambda_z / \Lambda_y\ ,\ L_p = \frac{\mu_0 l_x l_z}{\Lambda_y}\ ,\ M = \frac{\mu_0 l_x l_z}{\Lambda_y}$$

$$Q = \frac{\omega L_p}{R_p} = \frac{4 l_x l_z w}{\Lambda_y (l_x + l_z)\delta}\ ,$$

and C_p is the value of loop capacitor.

B. Dielectric Property

In the transmission line model, one has to also consider the coupling capacitor C_c existing between the wire loops and the conductors of transmission line, which has the significant effect on the dielectric property of material. The amount of the capacitor C_c can be estimated using the available analytical formulation for the capacitor per unit length between the co-planar strips [2]; and based on that the effective permittivity ε_{eff} of the equivalent medium is obtained to be

$$\varepsilon_{eff} = \varepsilon \left[1 + \frac{\Lambda_z l_x}{\Lambda_x \Lambda_y} \frac{K(\sqrt{1-g^2})}{K(g)} \right] \tag{2}$$

where K is the complete elliptic integral function and $g = w/(w+h)$.

This document has been submitted to the US patent office, Oct. 2002.

Therefore, a composite meta-material constructed of the proposed tank circuits presents a homogeneous anisotropic medium with an effective permittivity tensor having $\varepsilon_z = \varepsilon_{eff}$, $\varepsilon_x = \varepsilon_y = \varepsilon_0$, and effective permeability tensor having $\mu_y = \mu_{eff}$, $\mu_x = \mu_z = \mu_0$.

II. Electro-Ferromagnetism

The value of μ_{eff} of the circuit-embedded medium at a particular frequency depends on the resonant frequency of the embedded loops. Hence if the resonant frequency is changed, say by varying the loop capacitance C_p, the equivalent permeability of the medium can be varied. Of course changing C_p mechanically is not easy nor is it desirable. The application of electronic tunable capacitors seems to be an appropriate choice to make the medium electronically tunable.

Thin films of Barium-Strontium-Titanate ($Ba_xSr_{1-x}TiO_3$) possesses a high dielectric constant and ferro-electric properties [3]. This compound when used as a thin film in a capacitor (either in parallel plate or interdigitated configurations) produces an electrically small varactor with a relatively high tunability and high *Q*, while requiring a relatively low tuning voltage.

The BST varactors in each loop can simply be tuned by establishing a DC electric field in the medium. In order to tune the BST varactors by an applied electric field and design an electro-ferromagnetic tunable material, we need to modify the embedded circuit slightly. At DC, the loop varactor C_p is short-circuited, and therefore, the applied DC electric field will not be able to change the capacitance. However, if we place two series capacitors one on each side of the loop (as shown in Fig. 2), this problem can be circumvented.

As an example, consider a slab of the electro-ferromagnetic material confined between two parallel plates with a DC potential difference V_0. If there are N vertical loop layers between the plates, a voltage drop of V_0/N is experienced across a single layer. Referring to Fig. 2, it is now obvious that the tuning voltage across the varactors is simply given by

$$V_t = \frac{C_c}{C_c + 2C_p} \cdot \frac{V_0}{N}. \quad (3)$$

In practice, manufacturing of electro-ferromagnetic (tunable medium) embedded circuit meta-material can be simply performed using a stack of periodically printed circuits on a low-loss dielectric material. The loop capacitor can also be printed on the substrate, using simple gaps or interdigitated lines depending on the required values of capacitance.

III. FDTD Analysis

To successfully characterize the periodic meta-material and validate the analytical formulations derived based on the transmission line approach, a powerful and advanced computational tool based on the Finite Difference-Time Domain (FDTD) technique with Periodic Boundary Conditions/Perfectly Matched Layers (PBC/PML) [4] is used. The Prony method is also integrated to expedite the computational time. Taking advantage of the broadband analysis of FDTD provides a great efficiency in obtaining the wideband performance of complex structure.

IV. Performance Of Meta-Material

In order to examine the accuracy of the analytical formulation the FDTD with PBC/PML boundary conditions is applied to investigate the transmission coefficient of a normal incident plane wave through a slab of the circuit-embedded medium shown in Fig. 3(a). Fig. 4 compares the transmission coefficient calculated using FDTD for the embedded circuit medium and that of a homogeneous magneto-dielectric slab with thickness $t = 9.90\,mm$, relative effective permittivity $\varepsilon_{eff,r} = 10.89$, and relative effective permeability (plotted in Fig. 3(b))

$$\mu_{eff,r} = 1 - \frac{(0.49)^2}{1 - (3.25/f)^2}. \quad (4)$$

The excellent agreement between the analytical formulation and FDTD result is demonstrated.

The idea of embedded circuit meta-material can be extended to include dissimilar circuits. For example, Fig. 5(a) shows a periodic circuit-embedded medium where the odd and even layers have different loop capacitors. Since each circuit has a different resonant frequency the effective permeability of the medium has two distinct poles given by (Fig. 5(b))

$$\mu_{eff,r} = 1 - \frac{1}{2}\left(\frac{(0.49)^2}{1 - (2.58/f)^2} + \frac{(0.49)^2}{1 - (3.25/f)^2} \right). \quad (5)$$

The transmission coefficient calculated by the FDTD for the embedded-circuit material and that obtained for the equivalent slab are shown in Fig. 6 and illustrate the very good agreement.

To obtain an isotropic embedded circuit meta-material a 3-D tank circuit medium can be used.

References

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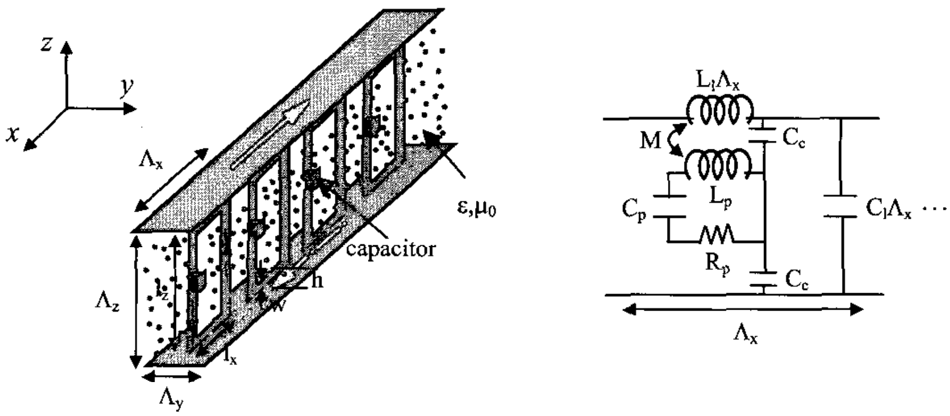


(a) (b)

Fig. 1: Periodically loaded transmission line by the thin wire loop tank circuits, (a) Building block unit cell, (b) Equivalent circuit.

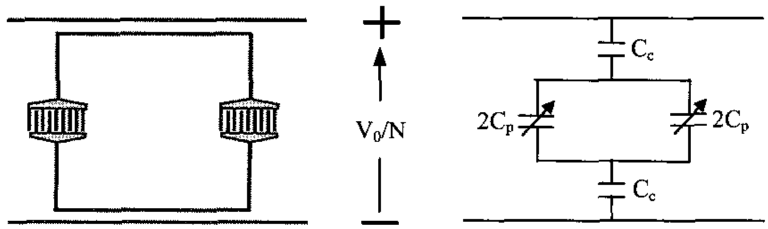
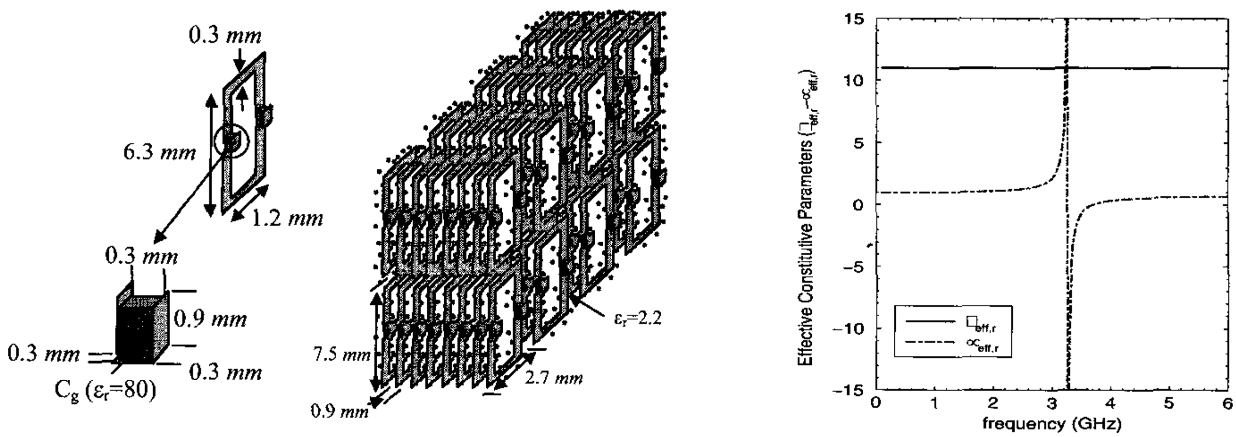
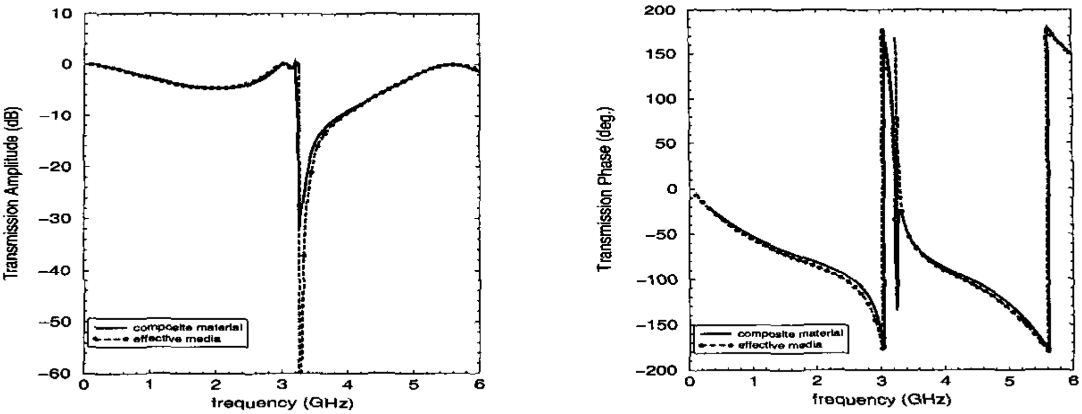


Fig. 2: Wire loop with tunable BST on both sides of the loop.



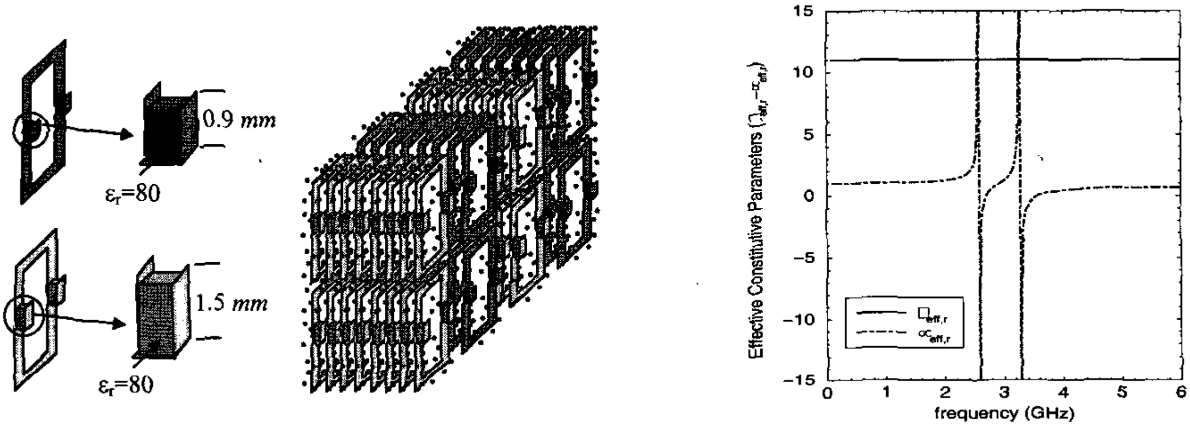
(a) (b)

Fig. 3: Single resonance embedded circuit meta-material, (a) Periodic structure, (b) ε_{eff} - μ_{eff} behavior.



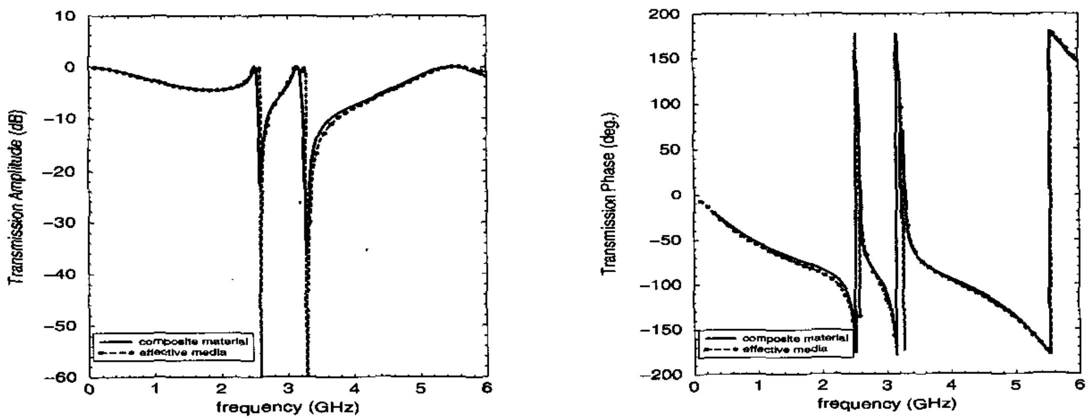
(a) (b)

Fig. 4: Performance of single resonance ε-μ meta-material, (a) Transmission amplitude, (b) Transmission phase.



(a) (b)

Fig. 5: Double resonance dissimilar embedded circuit meta-material, (a) Periodic structure, (b) ε_{eff} - μ_{eff} behavior.



(a) (b)

Fig. 6: Performance of double resonance ε-μ meta-material, (a) Transmission amplitude, (b) Transmission phase.